

Chimera-Like Coexistence of Synchronized Oscillation and Death in an Ecological Network

Partha Sharathi Dutta^{1,*} and Tanmoy Banerjee^{2,†}

¹*Department of Mathematics, Indian Institute of Technology Ropar, Rupnagar 140 001, Punjab, India.*

²*Department of Physics, University of Burdwan, Burdwan 713 104, West Bengal, India.*

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We report a novel spatiotemporal state, namely the chimera-like incongruous coexistence of *synchronized oscillation* and *stable steady state* (CSOD) in a realistic ecological network of nonlocally coupled oscillators. Unlike the *chimera* and *chimera death* state, in the CSOD state identical oscillators are self-organized into two coexisting spatially separated domains: In one domain neighboring oscillators show synchronized oscillation and in another domain the neighboring oscillators randomly populate either a synchronized oscillating state or a stable steady state (we call it a death state). We show that the interplay of nonlocality and coupling strength results in two routes to the CSOD state: One is from a coexisting mixed state of amplitude chimera and death, and another one is from a globally synchronized state. We further explore the importance of this study in ecology that gives a new insight into the relationship between spatial synchrony and global extinction of species.

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Understanding of collective dynamical behaviors in networks of coupled oscillators has been an active area of extensive research in the field of physics, chemistry, biology, engineering and social sciences. Coupled oscillators show a plethora of cooperative phenomena, such as synchronization [1], amplitude death [2], oscillation death [3], chimera [4], chimera death [5], etc. Two intriguing spatiotemporal dynamical states, namely the *chimera* and the recently observed *chimera death* have been in the center of recent research on coupled oscillators for their rich complex behaviors.

The chimera state is a fascinating spatiotemporal state where synchronous and asynchronous oscillations coexist in a network of coupled identical oscillators. After its discovery by Kuramoto and Battogtokh [6] and mathematical proof in Ref. [7], the chimera state attracts immediate attention due to its possible connection with unihemispheric sleep in certain species [4], the multiple time-scales of sleep dynamics [8], etc. Unlike phase chimera, where chimera occurs in the phase part, recently it is found that in the strong coupling limit amplitude effects come into play that results in amplitude mediated chimera [9] and amplitude chimera [5], [10]. The existence of chimera has also been established in many experiments, e.g., in optical system [11], chemical oscillators [12], mechanical system [13], and electronic system [14]. Further, chimera state has been observed in various fields; examples include [15]: FitzHugh-Nagumo oscillator, the SNIPER model of excitability of type-I, autonomous Boolean networks, etc (for an elaborate review please see [4]). Recently, chimera state in population dynamics is observed using Lattice Limit Cycle (LLC) model [16].

On the other hand, the *chimera death* (CD) state is discovered very recently by Zakharova *et al.* [5] in a network of Stuart-Landau oscillators under nonlocal cou-

pling. The CD state connects the chimera state to the oscillation death (OD) state [17]. In the OD state oscillators populate different branches of a stable inhomogeneous steady state (IHSS) [3, 17]. According to Ref.[5], in the *chimera death* state the population of oscillators in a network splits into coexisting domains of spatially coherent OD (where neighboring nodes attain essentially the same branch of the IHSS) and spatially incoherent OD (where the neighboring nodes jump among the different branches of IHSS in a completely random manner). Later, CD is also found in a network of mean-field diffusively coupled oscillators [18].

In summary, the *chimera* state is a spatial coexistence of coherent and incoherent **oscillations**, and the *chimera death* state is a spatial coexistence of coherent and incoherent branches of **oscillation death** state. Thus, the next natural question arises: *Is it possible to have an emergent state in a network of oscillators that shows a chimera-like coexistence of coherent oscillation and stable steady state?*

In this Letter, for the first time, we indeed find the answer in affirmative. Here, we address this open question and show that in a realistic ecological network consists of Rosenzweig–MacArthur oscillators [19] under nonlocal coupling topology, the interplay of non locality and coupling strength gives rise to a novel spatiotemporal state. In this state, the population of oscillators split into two coexisting distinct spatially separated domains: In one domain oscillators are oscillating in synchrony (i.e., coherently), and in another domain neighboring oscillators depict spatially synchronized oscillation and stable steady state in a random manner (i.e., incoherently). Hereafter, we call this hitherto unobserved state a *chimera-like synchronized oscillation and death* (CSOD) state (the stable steady state is denoted as a *death* state). Depending upon the coupling range

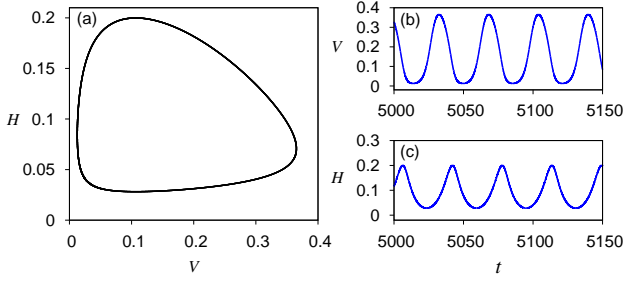


FIG. 1. (Color online) (a) Limit cycle attractor, and (b), (c) time-series of the uncoupled Rosenzweig–MacArthur model given by Eqs. (1) for $r = 0.5$, $K = 0.5$, $\alpha = 1$, $B = 0.16$, $\beta = 0.5$ and $m = 0.2$.

and coupling strength, we identify two types of transitions to the CSOD state: With increasing coupling range (for a moderate coupling strength) the CSOD state arises from a coexisting mixed state of amplitude chimera and death state; on the other hand, for an increasing coupling strength (with a moderate coupling range) the CSOD state comes from a globally synchronized oscillation state. However, in both the cases, under large coupling range and strength, the CSOD state is transformed into a chimera death state. *Thus, the CSOD state bridges the gap between the amplitude chimera and the chimera death state.* We further discuss the ecological importance of this emergent behavior that gives us a new insight into the relationship between spatial synchrony and global extinction of species, which are thought of as closely connected phenomena in ecology [20]: Spatial synchrony may lead to global extinction of species. Here we show that our results differ from this general consensus, and local extinction of a species does not necessarily lead to a global extinction of that species.

Here we consider the following network of N identical nonlocally coupled Rosenzweig–MacArthur (RM) oscillators:

$$\frac{dV_i}{dt} = rV_i \left(1 - \frac{V_i}{K}\right) - \frac{\alpha V_i}{V_i + B} H_i, \quad (1a)$$

$$\frac{dH_i}{dt} = H_i \left(\frac{\alpha \beta V_i}{V_i + B} - m \right) + \frac{\sigma}{2P} \sum_{k=i-P}^{i+P} (H_k - H_i), \quad (1b)$$

with V and H , respectively, representing vegetation and herbivore density, interacting in i ($i = 1, 2, \dots, N$) discrete patches (or nodes) (“ i ” is taken as modulo N). The local dynamics in each node are governed by the following system parameters: r is the intrinsic growth rate, K is the carrying capacity, α is the maximum predation rate of the herbivore, B is the half saturation constant, β represents the herbivore efficiency and m is the mortality rate of the herbivore. Interaction between nodes is governed by two coupling parameters: σ is the coupling strength and P controls the coupling range, where $1 \leq P \leq \frac{N}{2}$.

Two limiting values of P , i.e., $P = 1$ and $P = N/2$ represent local and global coupling, respectively. This nonlocal coupling topology has been used in Ref. [5] and Ref. [10] to observe chimera states in periodic and chaotic oscillators. The Rosenzweig–MacArthur model perhaps is the simplest model that can actually be applied in real ecosystems. As a result, this model becomes a standard spatially structured prey–predator model in theoretical ecology [21, 22].

An individual RM oscillator [see Eqs. (1) for $\sigma = 0$ and a fixed i] has the following steady states: (i) $(V^*, H^*) = (0, 0)$, the eigenvalues are $\lambda = r, -m$ and the equilibrium point is a saddle point, (ii) $(V^*, H^*) = (K, 0)$, the eigenvalues are $\lambda = -r, -m + \alpha \beta \frac{K}{K+B}$ and the equilibrium point is either a stable node or a saddle node, depending upon the values of the parameters, and finally (iii) $(V^*, H^*) = (\frac{mB}{\alpha\beta - m}, \frac{r}{\alpha}(1 - \frac{mB}{K(\alpha\beta - m)})(\frac{B\alpha\beta}{\alpha\beta - m}))$; this non-trivial equilibrium point is stable for parameter values satisfying the inequality $\frac{B}{K} < \frac{(\alpha\beta - m)}{(\alpha\beta + m)}$. Beyond a certain K , this equilibrium point becomes unstable and gives rise to a stable limit cycle through Hopf bifurcation. In general, further increase in K gradually increases amplitude of the limit cycle, thus bringing the density of either the prey or the predator or both the populations closer to zero, eventually leading to the extinction of the ecosystem; this is known as “the paradox of enrichment” [23] proposed by M. L. Rosenzweig in 1971. A subsequent realistic range [24] of K is 0.15 to 3 and range of m is 0.03 to 0.41. In Fig. 1, a stable limit cycle is shown for the following parameter values: $r = 0.5$, $K = 0.5$, $\alpha = 1$, $B = 0.16$, $\beta = 0.5$ and $m = 0.2$.

To explore various spatiotemporal patterns in the network, we take $N = 100$ and integrate Eqs. (1) numerically [25]. While presenting the simulation results, a large number of initial integration time ($t = 5000$) is discarded in order to ensure the steady state behavior. At first, we consider a moderate coupling strength, $\sigma = 1.7$, and increase the coupling range, $\gamma = P/N$, from a lower value. For lower coupling range ($\gamma \leq 0.05$) we observe a mixed state comprises of amplitude chimera and stable zero steady state (i.e., death state). This is shown in Figs. 2(a) and 2(b) for $\gamma = 0.01$ and $\gamma = 0.03$, respectively. The left panel in Fig. 2 shows the space-time color map of V_i and the right panel shows V_i in the steady state with oscillator index (“ i ”). The gray shaded regions in Figs. 2(a) and 2(b) (right panel) show this mixed state of amplitude chimera and death: *the amplitude chimera interrupted by death state is a new observation in the context of coupled oscillators.* Further increase in coupling range γ results in the CSOD state where the population of oscillators splits into two distinct coexisting domains: In one domain the neighboring nodes oscillate in synchrony while in another domain the neighboring nodes randomly populate either the synchronized oscillating state or the stable steady (death) state. This new spatiotemporal state

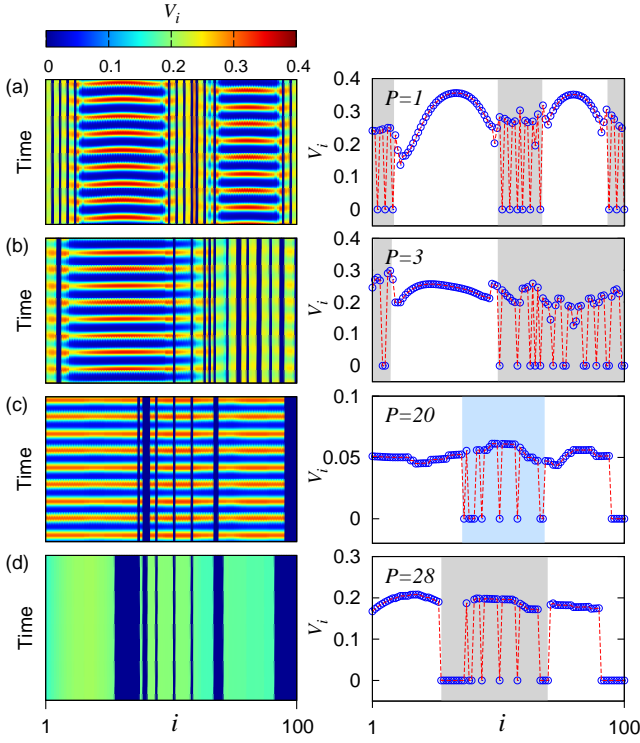


FIG. 2. (Color online) Left panel: Spatiotemporal color map and right panel: V_i with oscillator index “ i ” [red (dotted) line is for visual guidance]. Coupling strength $\sigma = 1.7$, $N = 100$. (a) $P = 1$ ($\gamma = 0.01$) and (b) $P = 3$ ($\gamma = 0.03$) show the mixed state of amplitude chimera and stable zero steady state (gray shade in right panels are for visual guidance). (c) $P = 20$ ($\gamma = 0.2$): *Chimera-like synchronized oscillation and death* (CSOD) state. Cyan (gray) shaded region in the right panel shows the random sequential occurrence of the synchronized oscillation and zero steady state of the neighboring nodes. (d) $P = 28$ ($\gamma = 0.28$): The chimera death state. Initial time $t = 5000$ is discarded before presenting the figures. Other parameters are same as used in Fig.1.

is shown in Fig. 2(c) for $\gamma = 0.2$ (i.e., $P = 20$). Here we see that in the shaded region [right panel of Fig. 2(c)] the neighboring oscillators populate either synchronized oscillation state or stable zero steady state in a random sequence. However, in the non shaded region only synchronized oscillation exists except in the rightmost nodes where few oscillators attain the death state. To the best of our knowledge, this chimera-like spatiotemporal state is new in literature because here stable steady state coexists with synchronized oscillations, which is unlike the chimera state or chimera death state. We also verify that this CSOD state is preserved for a larger number of nodes, N [26]. If we further increase the coupling range, we find the chimera death state [Fig. 2(d) for $\gamma = 0.28$]. We notice that instead of populating *two* branches of OD [5], [18], denoted as lower and upper branch, here in the CD state, the OD state has more than two branches [later it is clearly shown in Fig. 5(b)]. The multiple-branches

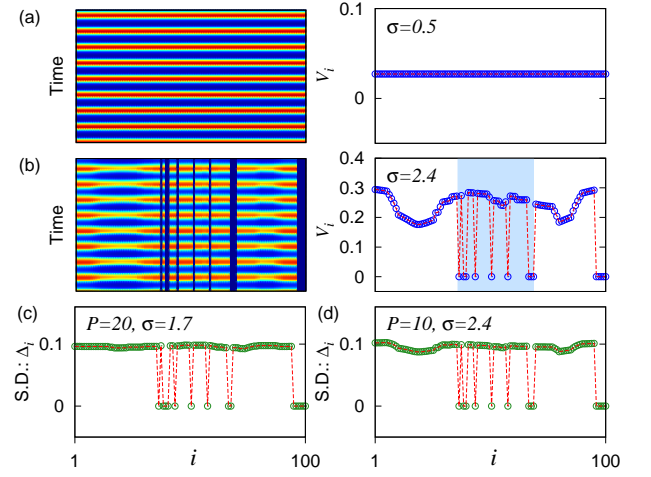


FIG. 3. (Color online) Left panel: Spatiotemporal color map (color bar same as Fig. 2), and right panel: V_i with oscillator index “ i ”. Coupling range $\gamma = 0.1$ (i.e., $P = 10$), $N = 100$. (a) $\sigma = 0.5$: Global in-phase synchronized oscillation. (b) $\sigma = 2.4$: The CSOD state. Cyan (gray) shaded region is for visual guidance of the incoherent region. (c) Standard deviation (S.D.) (Δ_i) with index “ i ” for the CSOD state of Fig. 2(c). (d) Δ_i with index “ i ” for the CSOD state of Fig. 3(b). Other parameters are same as used in Fig.1.

(more than two) of OD was reported earlier in [27] for sixteen *locally* coupled genetic relaxation oscillators, but in a network of large number of oscillators with *nonlocal* coupling it is an important observation (to be discussed later on).

We also identify one more significant route to the CSOD state with increasing coupling strength (σ) and a fixed γ , namely the transition from a global in-phase synchronized oscillating state to the CSOD state. This transition is shown in Figs. 3(a) and 3(b) for $\sigma = 0.5$ and $\sigma = 2.4$, respectively (for $\gamma = 0.1$, i.e., $P = 10$). Here also, the CSOD state is transformed into a multi-branch chimera death state for higher coupling strength (not shown).

Next, we quantify the spatiotemporal behavior where synchronized oscillation and stable zero steady (death) state coexist. In order to distinguish between the oscillation and death, we compute the standard deviation (S.D.) of each node given by:

$$\Delta_i = \sqrt{(\langle V_i^2 \rangle - \langle V_i \rangle^2)}. \quad (2)$$

The “ $\langle \rangle$ ” sign denotes the time average, which is carried out over a long time period ($t = 3000$ in the steady state). For a stable steady state (i.e., a death state) S.D. (Δ_i) must be zero and in the oscillatory condition it will show a finite non-zero value. Figures 3(c) and 3(d) show Δ_i values of the CSOD states shown in Fig. 2(c) and Fig. 3(b), respectively. For the CSOD state, in the incoherent region, we see that the Δ_i changes from a finite

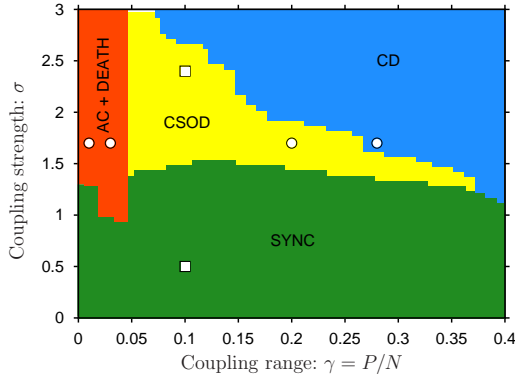


FIG. 4. (Color online) Phase diagram in the $\gamma - \sigma$ space. **CSOD**: chimera-like synchronized oscillation and stable zero steady state (death); **SYNC**: Global in-phase synchronized oscillation; **CD**: Chimera death; **AC+Death**: Coexistence of amplitude chimera and stable zero steady state. The symbols \circ indicate the coupling parameter values used for generating Figs. 2 (a)-2(d). \square represents the same for generating Figs. 3(a) and 3(b). Other parameters are same as used in Fig.1.

non-zero value to zero in a random manner; in the populations where nodes are oscillating in synchrony its value is non-zero and shows a continuous spatial variation.

In order to reveal the complete spatiotemporal scenario of the considered network, we rigorously compute the phase diagram in the $\gamma - \sigma$ space (the unsynchronized zone with very small σ value is not shown). From the phase diagram [Fig. 4] it is clear that the region of occurrence of the CSOD state is broad enough. It is seen that beyond $\gamma \approx 0.37$ (i.e., $P \approx 37$) no CSOD occurs; here an increase in σ transforms the synchronized oscillation state (SYNC) directly to the chimera death. The symbols \circ in the phase diagram indicate the coupling parameter values used for generating Figs. 2(a)-2(d), whereas \square represents the same for generating Figs. 3(a) and 3(b). In this context, it should be noted that in the phase diagram the boundaries among different phases are not very sharp, they tend to change with initial conditions. However, we observe that the overall qualitative structure of the phase diagram is preserved for all the initial conditions or number of nodes.

Next, we provide a qualitative explanation of the genesis of the CSOD state. We find that it has a strong connection with the *inhomogeneous limit-cycle* (IHLC) state in a network [28], [29]; IHLC is defined as a state where some nodes are in a stable steady state (or quasisteady state with a negligible amplitude, as in [29]), while the rest undergo oscillations. To visualize the scenario, we plot the time series of all the V_i 's [Fig. 5(a)] for an exemplary value of $\gamma = 0.2$ and $\sigma = 1.7$. From Fig. 5(a) one observes that a population of oscillators occupy the trivial zero steady state (i.e., $V = 0$ state), while the rest of the oscillators are in the in-phase synchronized oscillating state.

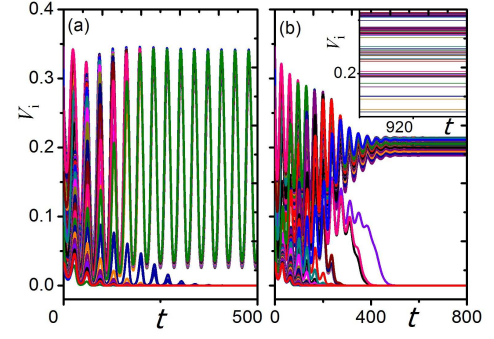


FIG. 5. (Color online) Time series of V_i 's for (a) $\gamma = 0.2$ and $\sigma = 1.7$: Synchronized limit-cycle oscillation (upper branch) and stable zero steady state (i.e., $V = 0$) coexists. (b) $\gamma = 0.28$ and $\sigma = 1.7$: Temporally stable multi-branch OD; Inset shows the multiple branches around $V_i = 0.2$. Other parameters are same as used in Fig.1.

Thus, depending upon judiciously chosen *spatial* initial conditions, individual nodes may populate either the upper oscillating branch or the lower steady state (i.e., $V = 0$) branch in a random sequence, which results in the CSOD state [as shown in Fig. 2(c)]. For higher coupling range and strength, e.g., $\gamma = 0.28$ and $\sigma = 1.7$, oscillators populate the multi-branch OD state [Fig. 5(b), see also the inset]; here a set of proper spatial initial conditions should result in chimera death in the network [as shown in Fig. 2(d)]. Thus, we may conjecture that, the CSOD state may occur in systems where this type of [Fig. 5(a)] IHLC state exists.

Finally, we discuss the importance of the results in ecology. In spatial ecology nonlocal coupling arises under the assumption that all spatially separated patches (or nodes) are connected only to certain number of neighboring nodes in a fragmented landscape, which is a more natural coupling scheme than the global coupling. In ecology, it is generally believed that spatial synchrony and global extinction are two strongly correlated phenomena (see for example, Refs.[20]). In contrast to this general belief, in the present study we show that, in non-local dispersive coupling, although spatial synchronization gives rise to local extinction of a species in one or more patches (or nodes) [i.e., the *death* state] but it defies the *global extinction* of the species (i.e., not all the oscillators go to the *death* state). Moreover, a general consensus in ecology is that, spatial synchrony and dispersal induced stability (or temporal stability) are two conflicting outcomes of dispersion among the population of patches. In the existing studies it is shown that dispersion among identical patches results in spatial synchrony; on the other hand, the combination of spatial heterogeneity and dispersion is necessary for dispersal-induced stability [30]. Here our results show that depending on coupling range and strength, spatial synchronization among *identical* patches (or nodes) leads to temporally stable

multi-branch (more than two) IHSS and a cluster of them has non-zero steady states. Thus, to achieve temporal stability the patches need not to be heterogeneous but nonlocal coupling is sufficient. Further, the occurrence of the amplitude chimera interrupted by death is a new finding and its proper interpretation in ecology deserves further attention.

In conclusion, in this Letter we have reported a novel spatiotemporal state, the CSOD state, in a realistic ecological network with nonlocal coupling topology. In this state a subset of oscillators populate spatially synchronized oscillation and stable steady state in a random manner, and the rest of the oscillators oscillate in synchrony. This spatiotemporal state is unlike the chimera state (where coherent and incoherent *oscillations* coexist) and the chimera death state (where neighboring oscillators populate two branches of *OD* in coherent and incoherent manner). We have shown two coupling dependent transition routes to this CSOD state. We further qualitatively established the connection of this emergent state with the inhomogeneous limit-cycle state present in the network. We have discussed the ecological importance of the results, which reveals that spatial synchrony does not necessarily lead to global extinction of a species, which is in contrast to the general consensus. Apart from ecology, we believe that, the present study will improve our understanding of other physical networks, e.g., power grid and communication networks, where it is desirable that a failure of certain nodes does not lead to a complete blackout or a global system failure [31].

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* parthasharathi@iitrpr.ac.in

† Corresponding author; tbanerjee@phys.buruniv.ac.in

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